Trinity College

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1,2
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number:　　In figures

In words　　_____

Your name　　_____

Time allowed for this section
Reading time before commencing work:　　ten minutes
Working time:　　one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items:　　pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:　　drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	101	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations.. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you do not use pencil, except in diagrams.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (101 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

(a) A body travels with a velocity $-35\mathbf{i} - 12\mathbf{j}$ ms^{-1}. Determine its speed and the bearing on which it is moving, assuming the positive y-axis to be due north. (3 marks)

(b) Given that $\lambda(7\mathbf{i} - 10\mathbf{j}) + \mu(-9\mathbf{i} + 14\mathbf{j}) = -31\mathbf{i} + 46\mathbf{j}$, determine the values of λ and μ. (4 marks)
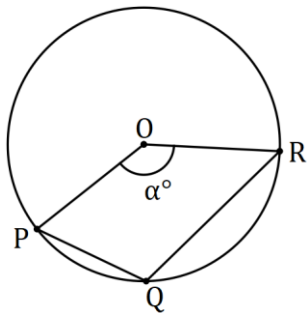
Question 10 **(6 marks)**

(a) In the diagram below, points A and B lie on a circle, DE is a tangent to the circle at C, $\angle DCA = 57°$ and $\angle CAB = 44°$. Determine the sizes of $\angle ABC, \angle BCE$ and $\angle BCA$. (3 marks)
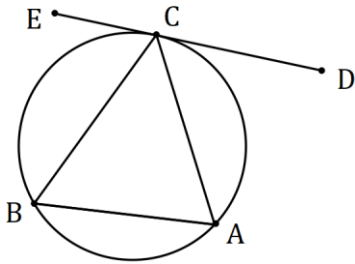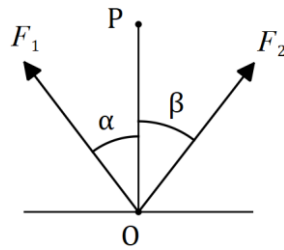

(b) In the next diagram, P, Q and R lie on a circle with centre O and $\angle POR = \alpha°$. Determine, with reasons, the size of $\angle PQR$ in terms of α. (3 marks)

See next page

Question 11   **(8 marks)**

Two forces, $F_1 = 550$ N and $F_2 = 770$ N, act on a body at O, and make angles of $\alpha = 33°$, and $\beta = 18°$ respectively with the vertical OP, as shown in the diagram below.

(a)   Determine the magnitude of the resultant force and the angle it makes with the vertical.

                                                                        (5 marks)

(b)   The magnitude of F_1 is to be adjusted so that the direction of the resultant is vertical. Determine the required magnitude of F_1.   (3 marks)

See next page

Question 12 **(7 marks)**

The largest Australian family recently met with the largest American family. Between them, these two families had 39 children.

(a)   Two of the children were chosen at random to feature in a TV documentary about the two families. Determine the number of different selections of two children that were possible.

(1 mark)

(b)   Prove that at least six of the children were born on the same day of the week.

(3 marks)

There were more children in the American family than the Australian family and the American children all had blue, brown or hazel coloured eyes.

(c)   Show that at least seven American children had the same eye colour.   (3 marks)

Question 13 **(11 marks)**

(a) Simplify $(5\mathbf{a} - 3\mathbf{b}) \cdot (2\mathbf{a} + 4\mathbf{b})$ given that $|\mathbf{a}| = 6, |\mathbf{b}| = 5$ and vector $\mathbf{a}$ is parallel and in the same direction to vector $\mathbf{b}$. (4 marks)

(b) With respect to the origin O, points A and B have position vectors $2\mathbf{i} + 3\mathbf{j}$ and $-\mathbf{i} + 5\mathbf{j}$. Determine $\overrightarrow{OC}$ if $\overrightarrow{AC} = 2\overrightarrow{AB}$. (4 marks)

(c) Points A, B and C have position vectors $(12, 7), (17, 3)$ and $(2, 15)$ respectively. Use a vector method to show that A, B and C are collinear. (3 marks)

Question 14   **(8 marks)**

(a)   Determine the size of angles x and y in the diagram below, where Q and S lie on the circumference of the circle with diameter PR.   (3 marks)
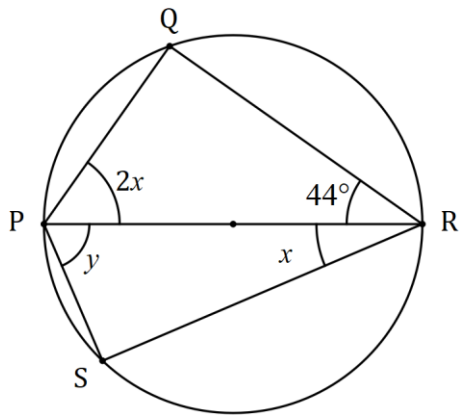
(b)   Triangle ABC has sides of length $AB = 4$ cm, $BC = 8$ cm and $AC = 7$ cm. Prove, using the method of contradiction, that if BC is a diameter of a circle then A does not lie on the circumference of the circle.   (5 marks)

Question 15 **(9 marks)**

(a) Determine the number of integers between 1 and 500 that are divisible by 6 or 7.

(4 marks)

(b) A pigeon fancier has 3 Fantail, 5 Carrier, 6 Archangel and 8 Dragoon pigeons and must choose four of them to enter in a local show. Determine the number of different ways the four pigeons can be chosen if
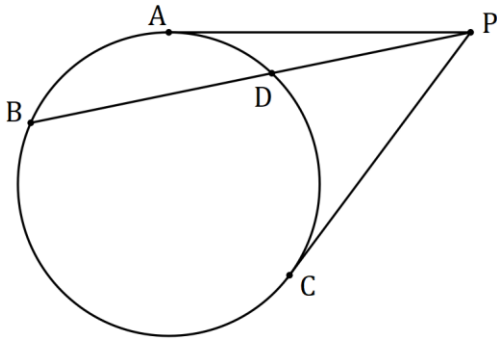
(i) there are no restrictions. (1 mark)

(ii) the fancier decides to take one of each breed. (2 marks)

(iii) the fancier decides to take at least three Carrier pigeons. (2 marks)
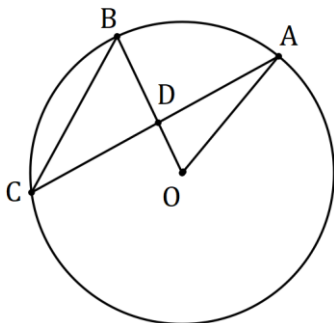
Question 16 **(9 marks)**

(a)   In the diagram below, PA and PC are tangents to the circle, with $PA = 58$ cm. Secant PB cuts the circle at D, so that $PD = 40$ cm. Determine the lengths of PC and BD.   (4 marks)

(b)   In the diagram below, A, B and C lie on the circumference of the circle with centre O, with AC intersecting OB at D. Prove that $\angle DAO = \angle DBC - \angle DCB$.   (5 marks)

See next page

Question 17 **(9 marks)**

Three vectors are $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$, $\mathbf{v} = \mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} - 3\mathbf{j}$.

(a) Determine the vector projection of $\mathbf{w}$ on $\mathbf{v}$ in exact form. (3 marks)

(b) Determine the scalar projection of $\mathbf{v}$ on $\mathbf{w}$. (2 marks)

(c) If $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and has the same magnitude as $\mathbf{w}$, determine the exact values of the coefficients a and b. (4 marks)

Question 18   **(7 marks)**

(a)   In the diagram below, points B and C lie on the minor arc AD of the circle with centre O.
      The lengths of chords AB and CD are congruent, $\angle BOC = 59°$ and $\angle AOD = 173°$.
      Determine the size of $\angle CBD$.   (3 marks)
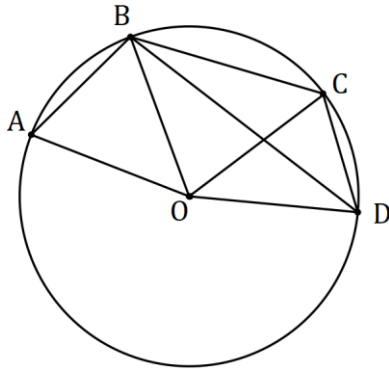
(b)   Line segment AC intersects line segment BD at N. Given that AC and BD are non-parallel
      and the lengths AN, AC, BN and BD are 35, 47, 59 and 67 cm respectively, explain
      whether the points A, B, C and D are concyclic.   (4 marks)

Question 19   **(6 marks)**

Daivik can row a boat at 2 m/s in still water. He sets out to cross a river which is running at 1.2 m/s.

(a)   In which direction should he row so that he crosses at right angles to the bank?   (3 marks)

(b)   Daivik decides to row so that he gets to the opposite bank as quickly as possible.

   (i)   What direction must he row in order to get to the opposite bank in the least time?

                                                                                       (1 marks)

   (ii)   If the river is 50 m wide, how long will it take to get to the other side?   (1 mark)

   (iii)   Where will Daivik land on the opposite bank?   (1 marks)

Question 20 **(7 marks)**

A small boat leaves jetty A to travel to jetty B, 377 m away on a bearing of 310°. A steady current of 1.5 ms^{-1} runs in the river between the jetties on a bearing 250°. If the small boat travels at a constant speed of 4.6 ms^{-1}, determine the bearing it should steer to reach jetty B and how long the journey will take.

Question 21 **(7 marks)**

A child is playing with thirteen coloured cubes, all the same size. There are six pink cubes, three navy and one each of red, blue, orange and green.

(a) If the child stacks cubes one on top of another to make a column, determine the number of different coloured columns that can be made using

(i) all the red, blue and green cubes. (1 mark)

(ii) all the pink, red and orange cubes. (2 marks)

(iii) all the cubes. (2 marks)

(b) If all but one of the cubes are used to make a column, determine the number of different coloured columns that can now be made. Justify your answer. (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____